

dependent on the service headway and the reliability of the departure time of the service to which passengers are incident.

After briefly introducing the random incidence model, which is often assumed to hold at short headways, the balance of this section reviews six studies of passenger incidence behavior that are motivated by understanding the relationships between service headway, service reliability, passenger incidence behavior, and passenger waiting time in a more nuanced fashion than is embedded in the random incidence assumption (2). Three of these studies depend on manually collected data, two studies use data from AFC systems, and one study analyzes the issue purely theoretically. These studies reveal much about passenger incidence behavior, but all are found to be limited in their general applicability by the methods with which they collect information about passengers and the services those passengers intend to use.

Random Passenger Incidence Behavior

One characterization of passenger incidence behavior is that of random incidence (3). The key assumption underlying the random incidence model is that the process of passenger arrivals to the public transport service is independent from the vehicle departure process of the service. This implies that passengers become incident to the service at a random time, and thus the instantaneous rate of passenger arrivals to the service is uniform over a given period of time. Let W and H be random variables representing passenger waiting times and service headways, respectively. Under the random incidence assumption and the assumption that vehicle capacity is not a binding constraint, a classic result of transportation science is that

$$E(W) = \frac{E[H^2]}{2E[H]} = \frac{E[H]}{2} (1 + CV(H)^2) \quad (1)$$

where $E[X]$ is the probabilistic expectation of some random variable X and $CV(H)$ is the coefficient of variation of H , a unitless measure of the variability of H defined as

$$CV(H) = \frac{\sigma_H}{E[H]} \quad (2)$$

where σ_H is the standard deviation of H (4). The second expression in Equation 1 is particularly useful because it expresses the mean passenger waiting time as the sum of two components: the waiting time caused by the mean headway (i.e., the reciprocal of service frequency) and the waiting time caused by the variability of the headways (which is one measure of service reliability). When the service is perfectly reliable with constant headways, the mean waiting time will be simply half the headway.

More Behaviorally Realistic Incidence Models

Jolliffe and Hutchinson studied bus passenger incidence in South London suburbs (5). They observed 10 bus stops for 1 h per day over 8 days, recording the times of passenger incidence and actual and scheduled bus departures. They limited their stop selection to those served by only a single bus route with a single service pattern so as to avoid ambiguity about which service a passenger was waiting for. The authors found that the actual average passenger waiting time was 30% less than predicted by the random incidence

model. They also found that the empirical distributions of passenger incidence times (by time of day) had peaks just before the respective average bus departure times. They hypothesized the existence of three classes of passengers: with proportion q , passengers whose time of incidence is causally coincident with that of a bus departure (e.g., because they saw the approaching bus from their home or a shop window); with proportion $p(1 - q)$, passengers who time their arrivals to minimize expected waiting time; and with proportion $(1 - p)(1 - q)$, passengers who are randomly incident. The authors found that p was positively correlated with the potential reduction in waiting time (compared with arriving randomly) that resulted from knowledge of the timetable and of service reliability. They also found p to be higher in the peak commuting periods rather than in the off-peak periods, indicating more awareness of the timetable or historical reliability, or both, by commuters.

Bowman and Turnquist built on the concept of aware and unaware passengers of proportions p and $(1 - p)$, respectively. They proposed a utility-based model to estimate p and the distribution of incidence times, and thus the mean waiting time, of aware passengers over a given headway as a function of the headway and reliability of bus departure times (1). They observed seven bus stops in Chicago, Illinois, each served by a single (different) bus route, between 6:00 and 8:00 a.m. for 5 to 10 days each. The bus routes had headways of 5 to 20 min and a range of reliabilities. The authors found that actual average waiting time was substantially less than predicted by the random incidence model. They estimated that p was not statistically significantly different from 1.0, which they explain by the fact that all observations were taken during peak commuting times. Their model predicts that the longer the headway and the more reliable the departures, the more peaked the distribution of incidence times will be and the closer that peak will be to the next scheduled departure time. This prediction demonstrates what they refer to as a safety margin that passengers add to reduce the chance of missing their bus when the service is known to be somewhat unreliable. Such a safety margin can also result from unreliability in passengers' journeys to the public transport stop or station. Bowman and Turnquist conclude from their model that the random incidence model underestimates the waiting time benefits of improving reliability and overestimates the waiting time benefits of increasing service frequency. This is because as reliability increases passengers can better predict departure times and so can time their incidence to decrease their waiting time.

Furth and Muller study the issue in a theoretical context and generally agree with the above findings (2). They are primarily concerned with the use of data from automatic vehicle-tracking systems to assess the impacts of reliability on passenger incidence behavior and waiting times. They propose that passengers will react to unreliability by departing earlier than they would with reliable services. Randomly incident unaware passengers will experience unreliability as a more dispersed distribution of headways and simply allocate additional time to their trip plan to improve the chance of arriving at their destination on time. Aware passengers, whose incidence is not entirely random, will react by timing their incidence somewhat earlier than the scheduled departure time to increase their chance of catching the desired service. The authors characterize these reactions as the costs of unreliability.

Luethi et al. continued with the analysis of manually collected data on actual passenger behavior (6). They use the language of probability to describe two classes of passengers. The first is timetable-dependent passengers (i.e., the aware passengers), whose incidence behavior is affected by awareness (possibly gained